

Group actions on Polish spaces

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In this paper we investigate the action of Polish groups (not necessarily abelian) on uncountable Polish spaces. We consider two main situations. First, when the orbits given by group action are small and the second when the family of orbits are at most countable. We have found some subgroups which are not measurable with respect to a given σ -ideal on the group and the action on some subsets gives a completely nonmeasurable set with respect to some σ -ideals with a Borel base on the Polish space. In most cases the general results are consistent with ZFC theory and are strictly connected with cardinal coefficients. We give some suitable examples, namely the subgroup of isometries of the Cantor space where the orbits are sufficiently small. In the opposite case we give an example of the group of the homeomorphisms of a Polish space in which there is a large orbit and we have found the subgroup without Baire property and a subset of the mentioned space such that the action of this subgroup on this set is completely nonmeasurable set with respect to the σ -ideal of the subsets of first category.

Presented results are obtained together with Szymon Żeberski..

REFERENCES

- [1] J. Cichoń, A. Kamburelis, J. Pawlikowski, On dense subsets of the measure algebra, Proc. Amer. Math. Soc. 94, 1421-146, (1985).
- [2] J. Cichoń, M. Morayne, R. Rałowski, Cz. Ryll-Nardzewski, Sz. Żeberski, On nonmeasurable unions, Topology Appl. 154, 884-893, (2007).
- [3] T. A. Chapman, Lectures on Hilbert cube manifolds, Amer Math. Soc., Providence, R. I., (1976).
- [4] T. Jech, Set theory, millenium edition, Springer Monographs in Mathematics, Springer-Verlag, (2003).
- [5] A. Kechris, Classical descriptive set theory, Springer, NY. (1995).
- [6] Z. Kostana, Non-meagre subgroups of reals disjoint with meagre sets. Topology Appl. 241, 11-19 (2018).
- [7] Y. Kuznetsova, On continuity of measurable group representations and homomorphisms, Stud. Mathematica, 210, 197-208 (2012).
- [8] R. Rałowski, P. Szczepaniak, Sz. Żeberski, A generalization of Steinhaus theorem and some nonmeasurable sets, Real Anall Exch. 35, 1-9 (2009/2010).
- [9] R. Rałowski, Sz. Żeberski, Completely nonmeasurable unions, Cent. Eur. J. Math. 8, 683-687 (2010).
- [10] R. Rałowski, Sz. Żeberski, On nonmeasurable images, Czechoslov. Math. J. 60, 424-434 (2010).
- [11] Sz. Żeberski, On completely nonmeasurable unions, Math. Log. Q. 53, 38-42 (2007).
- [12] Sz. Żeberski, Nonstandard proofs of Eggleston like theorems, Proceedings of the Ninth Prague Topological Symposium, Topol. Atlas, North Bay, (2002).

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